WNE Linear Algebra
Resit Exam
Series A

28 February 2020

Please use separate sheets for different problems. Please answer all questions on a single sheet. Give reasons to your answers. Please provide the following data on each sheet

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem and the series.**

Each problem is worth 10 marks and each question is worth 5 marks. To pass it necessary (but not sufficient) to score at least 50% out of 6 questions.

Problem 1.
Let $V = \text{lin}((1,3,10,11,4),(1,2,7,9,4),(3,-1,0,13,12))$ be a subspace of $\mathbb{R}^5$.

a) find a basis of the subspace V and the dimension of V,
b) find a system of linear equations which set of solutions is equal to V.

Problem 2.
Let $V \subset \mathbb{R}^5$ be a subspace given by the homogeneous system of linear equations

$$\begin{cases} x_1 + x_2 + 5x_3 + 4x_4 + 5x_5 = 0 \\ x_1 + 2x_2 + 4x_3 + 9x_4 + 10x_5 = 0 \end{cases}$$

a) find a basis $\mathcal{A}$ of the subspace V and the dimension of V,
b) extend the basis $\mathcal{A}$ to a basis $\mathcal{B}$ of $\mathbb{R}^5$ and find coordinates of vector $w = (-5,-4,1,1,0) \in \mathbb{R}^5$ relative to the basis $\mathcal{B}$.

Problem 3.
Let

$$A = \begin{bmatrix} -5 & 3 \\ -6 & 4 \end{bmatrix}.$$

a) find a matrix $C \in M(2 \times 2; \mathbb{R})$ such that

$$C^{-1}AC = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix},$$

where $a < 0$,
b) compute A^{35}.

Problem 4.
Let

$$A = \begin{bmatrix} 1 & 2t & 2 \\ t & 1 & t \\ 2 & t & 2 \end{bmatrix}.$$

a) for which $t \in \mathbb{R}$ is the matrix $A^2(A^\top)^3A^4$ invertible?
b) find the entry in the third row and the second column of the matrix A^{-1} for $t = 2$.

Problem 5.
Consider the following linear programming problem $-x_2 - 2x_3 + 3x_4 \to \min$ in the standard form with constraints

$$\begin{cases} \phantom{x_1 +{}} 2x_2 + x_3 \phantom{{}- x_4} = 1 \\ x_1 + x_2 \phantom{{}+x_3} - x_4 = 1 \\ - x_2 \phantom{{}+x_3} + x_4 = 1 \end{cases} \quad \text{and } x_i \geqslant 0 \text{ for } i = 1, \ldots, 4$$

a) which of the sets $\mathcal{B}_1 = \{1,3,4\}$, $\mathcal{B}_2 = \{1,2,3\}$, $\mathcal{B}_3 = \{2,3,4\}$ are basic? Which basic set is basic feasible? Write the corresponding feasible solution.
b) solve the linear programming problem using simplex method.

Questions

Question 1.
Is matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ positive definite for all $a, b, c, d > 0$?

Question 2.
If $v = (1,1,0), w = (-1,1,2) \in, \mathbb{R}^3,\ V = \operatorname{lin}(v)$, is the image of vector $w \in \mathbb{R}^3$ under the (linear) orthogonal symmetry about the subspace $V \subset \mathbb{R}^3$ equal to

$$S_V(w) = (1,-1,-2)?$$

Question 3.
If $A \in M(2 \times 2; \mathbb{R})$ and $A + A^\intercal = \mathbf{0}$, does it follow that $A^\intercal A = AA^\intercal$?

Question 4.
Is it possible that $A, B \in M(2 \times 2; \mathbb{R})$, $\det(A^2 + 2AB) \neq 0$ and $\det A = 0$?

Question 5.
Is it possible that $\mathcal{A}, \mathcal{B}$ are two bases of $\mathbb{R}^2$ and

$$M(\mathrm{id})_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}?$$

Question 6.
Are the affine subspaces $E,\ H \subset \mathbb{R}^3$ given by

$$E\colon \begin{cases} x_1 - x_2 \phantom{{}- x_3} = 5 \\ \phantom{x_1 -{}} 2x_2 - x_3 = 6 \end{cases},$$

$$H = (-1,0,2) + \operatorname{lin}((1,1,2)),$$

parallel?